The Linewidth of Ramsey Laser with Bad Cavity

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(Dated: January 15, 2010)

We investigate a new laser scheme by using Ramsey separated-field technique with bad cavity. By studying the linewidth of the stimulated-emission spectrum of this kind of laser inside the cavity, we find its linewidth is more than two orders of magnitude narrower than atomic natural linewidth, and it is far superior to that of conventional optical Ramsey method and any other available subnatural linewidth spectroscopy at present. Since any cavity related noise is reduced to cavity-pulling effect in bad cavity laser, this Ramsey laser provides the possibility of precision subnatural linewidth spectroscopy, which is critical for the next generation of optical clock and atom interferometers.

PACS numbers: 42.55.Ah, 42.50.Ar, 42.60.Da, 32.30.-r

Introduction: Since the invention of the separated-field technique [1], it has played an important role in the field of precision spectroscopy due to its linewidth narrowing effect via multiple coherent interaction. Atomic clocks based on this technique have greatly extended our ability for frequency measurement, further, almost all the atom interferometers are based on this technique [2].

Though, the natural linewidth of quantum transition was regarded as the ultimate limit to high-resolution laser spectroscopy [4], several methods of subnatural linewidth spectroscopy have been proposed to gain subnatural linewidth [3–10]. However, in all these efforts, including optical Ramsey spectroscopy, subnatural line is realized at the expense of a quick reduction in signal-to-noise (SNR) ratio due to the exponential decaying of signal, thus all these schemes can only get the linewidth several times narrower than the atomic natural linewidth. In the past three decades, this situation does not change in the field of the precision laser spectroscopy. On the other hand, the thermal noise of the cavity mirrors is the main obstacle for further linewidth reduction of a laser [11, 12], and it is a challenge to substantially reduce this noise further[13]. Recently, a new scheme, called active optical clock [14–18], was proposed to substantially reduce the laser linewidth. With lattice trapped atoms, it is possible to reach mHz linewidth laser based on the mechanism of active optical clock [14, 15, 19]. The principal mechanism of active optical clock is to directly extract light emitted from the ultranarrow atomic transition with a cavity mode linewidth much wider than that of lasing. This bad cavity ensures that any frequency shift due to cavity noise reduces to cavity-pulling effect [15–17], then the thermal noise is not the major obstacle again for reducing the linewidth. This means the bad cavity can play an indispensable role in new subnatural linewidth spectroscopy.

In this Letter, we propose a new scheme called Ramsey laser with bad cavity. Distinct from any previous applications of conventional Ramsey separated oscillating fields method [1], which focuses on the absorption spectrum, we here focus on the stimulated emission spectrum via multiple coherent interactions inside the cavity. We find this Ramsey laser can provide a stimulated-emission spectrum with a linewidth much narrower than that of any conventional optical Ramsey seperated-field spectroscopy, which is commonly applied in optical atomic clock. Our results also show that a subnatural linewidth spectroscopy, superior to any other available subnatural spectroscopy technique at present [3–10], can be reached by this kind of laser, if a suitable atomic level structure is chosen. Thus, this method can provide an effective subnatural spectroscopy, and the possibilities for the new optical clock scheme [15] and atom interferometers [2].

Theoretical framework: We consider the case of a two-level atomic beam interacting with a single-mode Ramsey cavity of separated-oscillating-field resonators with the cavity mode linewidth is much wider than the atomic gain linewidth. Thus we call it bad-cavity Ramsey laser. All atoms are pumped onto the upper lasing state **a** before entering the first cavity of seperated field, and the lower lasing state is **b**. We assume all the atoms have the same velocities v, that means what we consider here is a homogeneous laser system. And for the sake of simplicity, we consider the two-standing waves linear optical Ramsey configuration with a grid as spatial selector [20, 21]. Our treatment can be extended to other configurations as in [22–24]. The length of each oscillating part is l, and the length of the free drift region is L. The corresponding Hamiltonian is

$$\begin{aligned} H &= \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\sum_j[\omega_a^j(t)\sigma_a^j + \omega_b^j(t)\sigma_b^j] \\ &+ \hbar g\sum_j\Gamma_j(t)(\hat{a}^\dagger\hat{\sigma}_-^j e^{-i\vec{k}\cdot\vec{r}_j} + \hat{\sigma}_+^j\hat{a}e^{i\vec{k}\cdot\vec{r}_j}), \end{aligned} \quad (1)$$

where $\hat{a}$, $\hat{a}^\dagger$ are the annihilation and creation operators of the field mode inside the cavity, with the frequency ω, $\sigma_a^j = (|a\rangle\langle a|)^j$ and $\sigma_b^j = (|b\rangle\langle b|)^j$ are the projection operators for the jth atom corresponding to the upper and lower lasing levels,

with frequency ω_a^j and ω_b^j, and $\sigma_-^j = (|b\rangle\langle a|)^j$ is the "spin-flip" operator for the jth atom, with its adjoint $\sigma_+^j = (|a\rangle\langle b|)^j$. The coupling constant g is given by $g = \mu\sqrt{\omega/2\hbar\epsilon_0 V}$, where μ is the magnitude of the atomic dipole moment, and V is the effective volume of the cavity.

In order to denote the finite-time interaction between the atoms and Ramsey separated field, we introduce the function

$$\Gamma_j(t) = \Theta(t-t_j) - \Theta(t-t_j-\tau) + \Theta(t-t_j-\tau-T) - \Theta(t-t_j-2\tau-T), \tag{2}$$

where $\Theta(t)$ is the Heaviside step function [$\Theta(t) = 1$ for $t > 0$, $\Theta(t) = 1/2$ for $t = 0$, and $\Theta(t) = 0$ for $t < 0$]. T is the free drift time of the atoms, and τ is the interacting time between the atom and one cavity.

By the standard way [25], we can get the Heisenberg-Langevin equations of the motion for the single-atom and filed operators. By introducing the macroscopic atomic operator, $M(t) = -i\sum_j \Gamma_j(t)\sigma_-^j(t)$, $N_a(t) = \sum_j \Gamma_j(t)\sigma_{aa}^j(t)$, $N_b(t) = \sum_j \Gamma_j(t)\sigma_{bb}^j(t)$, the dynamic equations for the field and macroscopic atomic operators yield

$$\dot{a}(t) = -\frac{\kappa}{2}a(t) + gM(t) + F_\kappa(t), \tag{3}$$

$$\begin{aligned}\dot{N}_a(t) = \ & R(1 - A_0 + A_1 - A_2) - (\gamma_a + \gamma_a')N_a(t) \\ & - g[M^\dagger(t)a(t) + a^\dagger(t)M(t)] + F_a(t),\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{N}_b(t) = \ & -R(B_0 - B_1 + B_2) - \gamma_b N_b(t) + \gamma_a' N_a(t) \\ & + g[a^\dagger(t)M(t) + M^\dagger(t)a(t)] + F_b(t),\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{M}(t) = \ & -R(C_0 - C_1 + C_2) - \gamma_{ab}M(t) \\ & + g[N_a(t) - N_b(t)]a(t) + F_M(t),\end{aligned} \tag{6}$$

where the macroscopic noise operators are defined as

$$F_a(t) = \sum_j \dot{\Gamma}_j(t)\sigma_a^j(t) - R(1 - A_0 + A_1 - A_2) + \sum_j \Gamma_j(t)f_a^j(t),$$

$$F_b(t) = \sum_j \dot{\Gamma}_j(t)\sigma_b^j(t) + R(B_0 - B_1 + B_2) + \sum_j \Gamma_j(t)f_b^j(t),$$

$$F_M(t) = -i\sum_j \dot{\Gamma}_j(t)\tilde{\sigma}_-^j(t) + R(C_0 - C_1 + C_2) - i\sum_j \Gamma_j(t)f_\sigma^j(t),$$

with $A_0 = \left\langle\sigma_a^j(t_j+\tau)\right\rangle_q$, $A_1 = \left\langle\sigma_a^j(t_j+\tau+T)\right\rangle_q$, $A_2 = \left\langle\sigma_a^j(t_j+2\tau+T)\right\rangle_q$, $B_0 = \left\langle\sigma_b^j(t_j+\tau)\right\rangle_q$, $B_1 = \left\langle\sigma_b^j(t_j+\tau+T)\right\rangle_q$, $B_2 = \left\langle\sigma_b^j(t_j+2\tau+T)\right\rangle_q$, $C_0 = \left\langle-i\sigma_-^j(t_j+\tau)\right\rangle_q$, $C_1 = \left\langle-i\sigma_-^j(t_j+\tau+T)\right\rangle_q$, $C_2 = \left\langle-i\sigma_-^j(t_j+2\tau+T)\right\rangle_q$. R is the mean pumping rate, which is defined in [26]. It is very easy to check that the average values of the above Langevin forces are all zero.

By using the above definitions of the noise operators, we find the correlation functions of macroscopic noise forces can be generally written in the form

$$\begin{aligned}&\langle F_k(t)F_l(t')\rangle \\ = \ & D_{kl}^{(0)}\delta(t-t') + D_{kl}^{(1)}\delta(t-t'-\tau) \\ & + D_{kl}^{(2)}\delta(t-t'+\tau) + D_{kl}^{(3)}\delta(t-t'-\tau-T) \\ & + D_{kl}^{(4)}\delta(t-t'+\tau+T) + D_{kl}^{(5)}\delta(t-t'-2\tau-T) \\ & + D_{kl}^{(6)}\delta(t-t'+2\tau+T) + D_{kl}^{(7)}\delta(t-t'-T) \\ & + D_{kl}^{(8)}\delta(t-t'+T),\end{aligned} \tag{7}$$

where $D_{kl}^{(i)}(k,l = a,b,M,M^\dagger; i = 0,1,2)$ are the quantum diffusion coefficients.

c-number correlation functions: By choosing some particular ordering for products of atomic and field operators, one could derive the c-number stochastic Langevin equations from the quantum Langevin equations derived above, and all of the dynamic equations for c-number stochastic variables are the same as in [26]. The differences are from the correlation functions. On the other hand, we convert the quantum noise operators into the c-number noise variables $\tilde{F}_k(t)(k = a,b,M,M^\dagger)$, whose correlation functions are expressed as

$$\begin{aligned}&\langle\tilde{F}_k(t)\tilde{F}_k(t')\rangle \\ = \ & \tilde{D}_{kl}^{(0)}\delta(t-t') + \tilde{D}_{kl}^{(1)}\delta(t-t'-\tau) \\ & + \tilde{D}_{kl}^{(2)}\delta(t-t'+\tau) + \tilde{D}_{kl}^{(3)}\delta(t-t'-\tau-T) \\ & + \tilde{D}_{kl}^{(4)}\delta(t-t'+\tau+T) + \tilde{D}_{kl}^{(5)}\delta(t-t'-2\tau-T) \\ & + \tilde{D}_{kl}^{(6)}\delta(t-t'+2\tau+T) + \tilde{D}_{kl}^{(7)}\delta(t-t'-T) \\ & + \tilde{D}_{kl}^{(8)}\delta(t-t'+T),\end{aligned} \tag{8}$$

where $\tilde{D}_{kl}^{(i)}$ are the c-number Langevin diffusion coefficients, related to quantum Langevin diffusion coefficients $D_{kl}^{(i)}$ as in [27].

Steady-state solutions: The steady-state solutions for the mean values of the field and atomic variables for laser operation are obtained by dropping the noise terms of the c-number Langevin equations and setting the time derivatives equal to zero. The analytical solutions are very complex, and one could numerically solve the steady-state equations. In this paper, we only care about the bad cavity limit $\gamma_{max} \ll T^{-1} \ll \tau^{-1} \ll \kappa/2$. Since the atomic transit time is much shorter than the damping times of atomic variables, one could ignore the effect of the spontaneous emission of the atom. By the standard way [25], We get the following steady-state values:

$$\left|\tilde{A}_{ss}\right|^2 = \frac{R(1 - A_0 + A_1 - A_2)}{\kappa} = \frac{R(B_0 - B_1 + B_2)}{\kappa},$$

$$\tilde{N}_{ass} = \frac{R\tau}{2}\left[1 + \frac{C_0 - C_1 + C_2}{g\tau}\sqrt{\frac{\kappa}{R(B_0 - B_1 + B_2)}}\right],$$

$$\tilde{N}_{bss} = \frac{R\tau}{2}\left[1 - \frac{C_0 - C_1 + C_2}{g\tau}\sqrt{\frac{\kappa}{R(B_0 - B_1 + B_2)}}\right].$$

A detailed analysis about the stability of the steady-state can be found such as in [28]. In this paper, we assume the steady-state solution is stable.

Laser linwidth: Suppose the quantum fluctuation is small, the evolution of the fluctuations can be obtained by making a linearization of the c-number Langevin equations around the steady-state solution. Then the measured spectra of field fluctuations will be directly related to these quantities. By Fourier transformations of the linearized equation, we get the amplitude and phase quadrature components $\delta X(\omega)$ and $\delta Y(\omega)$ [26]. Well above threshold, one can neglect the amplitude fluctuations, and the linewidth inside the cavity is related to the phase-diffusion coefficient [25]. For small fluctuation of laser phase, the spectrum of phase fluctuations is simply related to the spectrum of the phase quadrature component of the field fluctuations, namely,

$$(\delta\varphi^2)_\omega = \frac{1}{I_0}(\delta Y^2)_\omega.$$

In the region $\gamma_{ab} \ll T^{-1} \ll \tau^{-1} \ll \kappa/2$, as in the recently proposed active optical clock [15] with atomic beam. The phase quadrature component of the field fluctuations can be expressed as

$$
\begin{aligned}
(\delta\varphi^2)_\omega \\
\approx \frac{(\kappa/2 + \gamma_{ab})^2}{I_0\omega^2[(\kappa/2 + \gamma_{ab})^2 + \omega^2]}\frac{g^2}{4(\kappa/2 + \gamma_{ab})^2}\{4\gamma_{ab}\tilde{N}_{ass} \\
+ 2R[(A_0 + B_0) + (A_2 + B_2)] \\
+ Rp[(C_0 - C_0^*)^2 + (C_1 - C_1^*)^2 + (C_2 - C_2^*)^2]\}.
\end{aligned}
\tag{9}
$$

Since the time τ and T is much shorter than the time scale of the atomic dampings, we can neglect the dampings when calculate A_i, B_i, C_i. By using

$$A_0 = \cos^2\left(\frac{\Omega_R}{2}\tau\right), \quad A_1 = \cos^2\left(\frac{\Omega_R}{2}\tau\right),$$

$$A_2 = 1 - \sin^2(\Omega_R\tau)\cos^2\left(\frac{\Delta_2}{2}T\right), \quad B_0 = \sin^2\left(\frac{\Omega_R}{2}\tau\right),$$

$$B_1 = \sin^2\left(\frac{\Omega_R}{2}\tau\right), \quad B_2 = \sin^2(\Omega_R\tau)\cos^2\left(\frac{\Delta_2 T}{2}\right),$$

$$(C_0 - C_0^*)^2 = 0, (C_1 - C_1^*)^2 = -\sin^2(\Omega_R\tau)\sin^2(\Delta_2 T),$$

$$(C_2 - C_2^*)^2 = -\sin^2(\Omega_R\tau)\sin^2(\Delta_2 T),$$

we get

$$
\begin{aligned}
(\delta\varphi^2)_\omega &= \frac{(\kappa/2 + \gamma_{ab})^2}{\omega^2[(\kappa/2 + \gamma_{ab})^2 + \omega^2]}\frac{\gamma_{ab}^2}{(\kappa/2 + \gamma_{ab})^2}\{D_{ST} \\
&+ D_{Ram}[2 - p\sin^2(\Omega_R\tau)\sin^2(\Delta_2 T)]\},
\end{aligned}
\tag{10}
$$

where Ω_R is the Rabi frequency on resonance, $D_{ST}=g^2\tilde{N}_{ass}/I_0\gamma_{ab}$, $D_{Ram} = g^2R/2I_0\gamma_{ab}^2$, and $\Delta_2 = \omega - (\omega_{a2} - \omega_{b2})$ presents the detuning in the free drift region. p is a parameter, which characterizes the pumping statistics: a Poissonian excitation statistics corresponds to $p = 0$, and for a regular statistics we have $p = 1$.

Then the linewidth of Ramsey laser with bad cavity is given by

$$D = \frac{\gamma_{ab}^2}{(\kappa/2 + \gamma_{ab})^2}\{D_{ST} + D_{Ram}[2 - p\sin^2(\Omega_R\tau)\sin^2(\Delta_2 T)]\}.
\tag{11}$$

Since $D_{ST}/D_{Ram} \ll 1$ in our situation, and in the case of maximal photon number, the steady state value of $\tilde{N}_{ass}$ is about $R\tau/2$. Then we get the

$$D \approx \frac{2g^2}{\kappa}[2 - p\sin^2(\Omega_R\tau)\sin^2(\Delta_2 T)].
\tag{12}$$

From the expression above, we find that the pumping statistic can influence the linewidth. For regular injection ($p = 1$), the linewidth is the narrowest, while for Poissonian injection ($p = 0$), the linewidth is the broadest. But even for regular injection, the linewidth is larger than the case of one cavity. That means the mechanism of separated-field does not play the role in reducing the linewidth as in the conventional optical Ramsey method, which is counter-intuitive. However, the separated fields are indispensable for any phase detection like atom interferometry. The details about the method of active atom interferometry will appear elsewhere.

Our method of Ramsey laser is suitable for any atoms with metastable energy level, as an example, we choose the transition from the metastable state $4s4p\ ^3P_1$ to the ground state $4s^2\ ^1S_0$ of ^{40}Ca to check the striking feature of this laser: subnatural linewidth. As mentioned in [29], the corresponding natural linewidth of the metastable state $4s4p\ ^3P_1$ is 320Hz. As in the recently proposed active optical clock with atomic beam [15], the velocity of the atoms in thermal atomic beam is about 500m/s, and the length of the interaction region is about 1mm, then the time for the atom to traverse each coherent-interaction region is on the order of magnitude of 1 μs. If a bad cavity with κ is on the order of 10^7Hz, the relation $\kappa/2 \gg \tau^{-1}$ is satisfied. Then when g is on the order of the magnitude of kHz, which can be easily achieved for current technique [30], from the linewidth expression of Eq.(16) the order of magnitude of linewidth is below 1 Hz. This means the linewidth of a Ramsey laser can be more than two orders of magnitude narrower than the atomic natural linewidth, therefore our Ramsey method provides a new subnatural spectroscopy technique. And since it is stimulated-emission spectrum, it overcomes the difficulty in other subnatural linewidth spectroscopy schemes where the quick reduction of signal to noise ratio is a formidable limit. We should point out that this Ramsey laser does not escape the limitation of all active optical clock: in order to pump atoms to the excited state effectively and to be stimulated emit photon during the lifetime of a metastable state, this new method will only be applicable to some special transitions [17].

Conclusion: In summary, we propose a new subnatural linewidth spectroscopy technique, which is a laser by using Ramsey seperated-field cavity to realize the output of stimulated-emission radiation via multiple coherent interaction with atomic beam. We find the linewidth of Ramsey laser is subnatural if we choose an appropriate atomic level, and the bad-cavity laser mechanism will dramatically reduce cavity-related noise as discussed in active optical clock [15–19]. Our results show that this new subnatural linewidth spectroscopy is superior to conventional optical Ramsey seperated-field spectroscopy and any other available subnatural spectroscopy technique at present [3–10]. Considering one have to apply the separated-field method in any phase detection as in Ramsey-Bordé interferometer [2], to investigate the effects of phase differences between the two oscillating fields [31] in this stimulated separated-field method with such subnatural linewidth will be our next research aim.

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